Space and Geometry

Topology and Geometry for Physicists

Kähler geometry is a beautiful and intriguing area of mathematics, of substantial research interest to both mathematicians and physicists. This self-contained graduate text provides a concise and accessible introduction to the topic. The book begins with a review of basic differential geometry, before moving on to a description of complex manifolds and holomorphic vector bundles. Kähler manifolds are discussed from the point of view of Riemannian geometry, and Hodge and Dolbeault theories are outlined, together with a simple proof of the famous Kähler identities. The final part of the text studies several aspects of compact Kähler manifolds: the Calabi conjecture, Weitzenböck techniques, Calabi-Yau manifolds, and divisors. All sections of the book end with a series of exercises and students and researchers working in the fields of algebraic and differential geometry and theoretical physics will find that the book provides them with a
sound understanding of this theory.

The Calabi-Yau Landscape

Although contact geometry and topology is briefly discussed in V I Arnol’d’s book *Mathematical Methods of Classical Mechanics* (Springer-Verlag, 1989, 2nd edition), it still remains a domain of research in pure mathematics, e.g. see the recent monograph by H Geiges *An Introduction to Contact Topology* (Cambridge U Press, 2008). Some attempts to use contact geometry in physics were made in the monograph *Contact Geometry and Nonlinear Differential Equations* (Cambridge U Press, 2007). Unfortunately, even the excellent style of this monograph is not sufficient to attract the attention of the physics community to this type of problems. This book is the first serious attempt to change the existing status quo. In it we demonstrate that, in fact, all branches of theoretical physics can be rewritten in the language of contact geometry and topology: from mechanics, thermodynamics and electrodynamics to optics, gauge fields and gravity; from physics of liquid crystals to quantum mechanics and quantum computers, etc. The book is written in the style of famous Landau-Lifshitz (L-L) multivolume course in theoretical physics. This means that its readers are expected to have solid background in theoretical physics (at least at the level of the L-L course). No prior knowledge of specialized mathematics is required. All needed new mathematics is given in the context of discussed physical problems. As in the L-L course some problems/exercises are formulated along the way and, again as in the L-L course, these are always supplemented by either solutions or by hints (with exact references). Unlike the L-L course, though, some definitions, theorems, and remarks are also presented. This is done with the purpose of stimulating the interest of our readers in deeper study of subject matters discussed in the text.

Differential Geometry and Lie Groups for Physicists

Although contact geometry and topology is briefly discussed in V I Arnol’d’s book “*Mathematical Methods of Classical Mechanics* ”(Springer-Verlag, 1989, 2nd edition), it still remains a domain of research in pure mathematics, e.g. see the recent monograph by H Geiges “*An Introduction to Contact Topology*” (Cambridge U Press, 2008). Some attempts to use contact geometry in physics were made in the monograph “*Contact Geometry and Nonlinear Differential Equations*” (Cambridge U Press, 2007). Unfortunately, even the excellent style of this monograph is not sufficient to attract the attention of the physics community to this type of problems. This book is the first serious attempt to change the existing status quo. In it we demonstrate that, in fact, all branches of theoretical physics can be rewritten in the language of contact geometry and topology: from mechanics, thermodynamics and electrodynamics to optics, gauge fields and gravity; from physics of liquid crystals to quantum mechanics and quantum computers, etc. The book is written in the style of famous Landau-Lifshitz (L-L) multivolume course in theoretical physics. This means that its readers are expected to have solid background in theoretical physics (at least at the level of the L-L course). No prior knowledge of specialized mathematics is required. All needed new mathematics is given in the context of discussed physical problems. As in the L-L course some problems/exercises are formulated along the way and, again as in the L-L course, these are always supplemented by either solutions or by hints (with exact references). Unlike the L-L course, though, some definitions, theorems, and remarks are also presented. This is done with the purpose of stimulating the interest of our readers in deeper study of subject matters discussed in the text. Contents:Motivation and BackgroundFrom Ideal Magnetohydrodynamics to String and Knot TheoryAll About and Around Woltjer’s TheoremTopologically Massive Gauge Theories and Force-Free FieldsContact Geometry and PhysicsSub-Riemannian Geometry, Heisenberg Manifolds and Quantum Mechanics of Landau LevelsAbrikosov Lattices, TGB Phases in Liquid
The Formative Years of Relativity

For physicists and applied mathematicians working in the fields of relativity and cosmology, high-energy physics and field theory, thermodynamics, fluid dynamics and mechanics. This book provides an introduction to the concepts and techniques of modern differential theory, particularly Lie groups, Lie forms and differential forms.

Lie Groups, Physics, and Geometry

These three essays by an eminent scientist explore the nature, origin, and development of our concepts of space from the points of view of the senses, history, and physics. They examine the subject from every direction, in a manner suitable for both undergraduates and other readers. 25 figures. 1906 edition.

Defects and Geometry in Condensed Matter Physics

Quantum information theory is a branch of science at the frontier of physics, mathematics, and information science, and offers a variety of solutions that are impossible using classical theory. This book provides a detailed introduction to the key concepts used in processing quantum information and reveals that quantum mechanics is a generalisation of classical probability theory. The second edition contains new sections and entirely new chapters: the hot topic of multipartite entanglement; in-depth discussion of the discrete structures in finite dimensional Hilbert space, including unitary operator bases, mutually unbiased bases, symmetric informationally complete generalized measurements, discrete Wigner function, and unitary designs; the Gleason and Kochen–Specker theorems; the proof of the Lieb conjecture; the measure concentration phenomenon; and the Hastings’ non-additivity theorem. This richly-illustrated book will be useful to a broad audience of graduates and researchers interested in quantum information theory. Exercises follow each chapter, with hints and answers supplied.
Advanced General Relativity

This richly illustrated and clearly written undergraduate textbook captures the excitement and beauty of geometry. The approach is that of Klein in his Erlangen programme: a geometry is a space together with a set of transformations of the space. The authors explore various geometries: affine, projective, inversive, hyperbolic and elliptic. In each case they carefully explain the key results and discuss the relationships between the geometries. New features in this second edition include concise end-of-chapter summaries to aid student revision, a list of further reading and a list of special symbols. The authors have also revised many of the end-of-chapter exercises to make them more challenging and to include some interesting new results. Full solutions to the 200 problems are included in the text, while complete solutions to all of the end-of-chapter exercises are available in a new Instructors’ Manual, which can be downloaded from www.cambridge.org/9781107647831.

The Geometry of Kerr Black Holes

This book provides advanced undergraduate physics and mathematics students with an accessible yet detailed understanding of the fundamentals of differential geometry and symmetries in classical physics. Readers, working through the book, will obtain a thorough understanding of symmetry principles and their application in mechanics, field theory, and general relativity, and in addition acquire the necessary calculational skills to tackle more sophisticated questions in theoretical physics. Most of the topics covered in this book have previously only been scattered across many different sources of literature, therefore this is the first book to coherently present this treatment of topics in one comprehensive volume. Key features: Contains a modern, streamlined presentation of classical topics, which are normally taught separately Includes several advanced topics, such as the Belinfante energy-momentum tensor, the Weyl-Schouten theorem, the derivation of Noether currents for diffeomorphisms, and the definition of conserved integrals in general relativity Focuses on the clear presentation of the mathematical notions and calculational technique

Geometry

Relativity and Geometry aims to elucidate the motivation and significance of the changes in physical geometry brought about by Einstein, in both the first and the second phases of relativity. The book contains seven chapters and a mathematical appendix. The first two chapters review a historical background of relativity. Chapter 3 centers on Einstein’s first Relativity paper of 1905. Subsequent chapter presents the Minkowskian formulation of special relativity. Chapters 5 and 6 deal with Einstein’s search for general relativity from 1907 to 1915, as well as some aspects and subsequent developments of the theory. The last chapter explores the concept of simultaneity, geometric conventionalism, and a few other questions concerning space time structure, causality, and time.

Mathematics for Physics

In recent years the methods of modern differential geometry have become of considerable importance in theoretical physics and have found application in relativity and cosmology, high-energy physics and field theory, thermodynamics, fluid dynamics and mechanics. This textbook provides an introduction to these methods - in particular Lie derivatives, Lie groups and differential forms - and covers their extensive
applications to theoretical physics. The reader is assumed to have some familiarity with advanced calculus, linear algebra and a little elementary operator theory. The advanced physics undergraduate should therefore find the presentation quite accessible. This account will prove valuable for those with backgrounds in physics and applied mathematics who desire an introduction to the subject. Having studied the book, the reader will be able to comprehend research papers that use this mathematics and follow more advanced pure-mathematical expositions.

**The Geometry of Physics**

An engagingly-written account of mathematical tools and ideas, this book provides a graduate-level introduction to the mathematics used in research in physics. The first half of the book focuses on the traditional mathematical methods of physics - differential and integral equations, Fourier series and the calculus of variations. The second half contains an introduction to more advanced subjects, including differential geometry, topology and complex variables. The authors' exposition avoids excess rigor whilst explaining subtle but important points often glossed over in more elementary texts. The topics are illustrated at every stage by carefully chosen examples, exercises and problems drawn from realistic physics settings. These make it useful both as a textbook in advanced courses and for self-study. Password-protected solutions to the exercises are available to instructors at www.cambridge.org/9780521854030.

**Principal Bundles**

Describes random geometry and applications to strings, quantum gravity, topological field theory and membrane physics.

**New Spaces in Physics: Volume 2**

**Publisher Description**

**Quantum Geometry**

Deals with the twistor treatment of certain linear and non-linear partial differential equations. The description in terms of twistors involves algebraic and differential geometry, and several complex variables.

**Geometrical Methods of Mathematical Physics**

Differential geometry and topology have become essential tools for many theoretical physicists. In particular, they are indispensable in theoretical studies of condensed matter physics, gravity, and particle physics. Geometry, Topology and Physics, Second Edition introduces the ideas and techniques of differential geometry and topology at a level suitable for postgraduate students and researchers in these fields. The second edition of this popular and established text incorporates a number of changes designed to meet the needs of the reader and reflect the development of the subject. The book features a considerably expanded first chapter, reviewing aspects of path integral
Partial Differential Equations arising from Physics and Geometry

After the development of manifolds and algebraic varieties in the previous century, mathematicians and physicists have continued to advance concepts of space. This book and its companion explore various new notions of space, including both formal and conceptual points of view, as presented by leading experts at the New Spaces in Mathematics and Physics workshop held at the Institut Henri Poincaré in 2015. This volume covers a broad range of topics in mathematical physics, including noncommutative geometry, supergeometry, derived symplectic geometry, higher geometric quantization, intuitionistic quantum logic, problems with the continuum description of spacetime, twistor theory, loop quantum gravity, and geometry in string theory. It is addressed primarily to mathematical physicists and mathematicians, but also to historians and philosophers of these disciplines.

Applications of Contact Geometry and Topology in Physics

A self-contained introduction to advanced general relativity.

Dirac Operators and Spectral Geometry

Applications from condensed matter physics, statistical mechanics and elementary particle theory appear in the book. An obvious omission here is general relativity—we apologize for this. We originally intended to discuss general relativity. However, both the need to keep the size of the book within the reasonable limits and the fact that accounts of the topology and geometry of relativity are already available, for example, in The Large Scale Structure of Space-Time by S. Hawking and G. Ellis, made us reluctantly decide to omit this topic.

Differential Geometry, Gauge Theories, and Gravity

Geometric algebra is a powerful mathematical language with applications across a range of subjects in physics and engineering. This book is a complete guide to the current state of the subject with early chapters providing a self-contained introduction to geometric algebra. Topics covered include new techniques for handling rotations in arbitrary dimensions, and the links between rotations, bivectors and the structure of the Lie groups. Following chapters extend the concept of a complex analytic function theory to arbitrary dimensions, with applications in quantum theory and electromagnetism. Later chapters cover advanced topics such as non-Euclidean geometry, quantum entanglement, and
gauge theories. Applications such as black holes and cosmic strings are also explored. It can be used as a graduate text for courses on the physical applications of geometric algebra and is also suitable for researchers working in the fields of relativity and quantum theory.

**Quantum Riemannian Geometry**

Geometry aims to describe the world around us. It is central to many branches of mathematics and physics, and offers a whole range of views on the universe. This is an introduction to the ideas of geometry and includes generous helpings of simple explanations and examples. The book is based on many years teaching experience so is thoroughly class-tested, and as prerequisites are minimal, it is suited to newcomers to the subject. There are plenty of illustrations; chapters end with a collection of exercises, and solutions are available for teachers.

**Geometry of Quantum States**

Einstein's General Theory of Relativity leads to two remarkable predictions: first, that the ultimate destiny of many massive stars is to undergo gravitational collapse and to disappear from view, leaving behind a 'black hole' in space; and secondly, that there will exist singularities in space-time itself. These singularities are places where space-time begins or ends, and the presently known laws of physics break down. They will occur inside black holes, and in the past are what might be construed as the beginning of the universe. To show how these predictions arise, the authors discuss the General Theory of Relativity in the large. Starting with a precise formulation of the theory and an account of the necessary background of differential geometry, the significance of space-time curvature is discussed and the global properties of a number of exact solutions of Einstein's field equations are examined. The theory of the causal structure of a general space-time is developed, and is used to study black holes and to prove a number of theorems establishing the inevitability of singularities under certain conditions. A discussion of the Cauchy problem for General Relativity is also included in this 1973 book.

**Geometry, Symmetries, and Classical Physics**

A clear, concise and up-to-date introduction to the theory of the Dirac operator and its wide range of applications in theoretical physics for graduate students and researchers.

**The Large Scale Structure of Space-Time**

A comprehensive survey of all the mathematical methods that should be available to graduate students in physics. In addition to the usual topics of analysis, such as infinite series, functions of a complex variable and some differential equations as well as linear vector spaces, this book includes a more extensive discussion of group theory than can be found in other current textbooks. The main feature of this textbook is its extensive treatment of geometrical methods as applied to physics. With its introduction of differentiable manifolds and a discussion of vectors and forms on such manifolds as part of a first-year graduate course in mathematical methods, the text allows students to grasp at an early stage the contemporary literature on dynamical systems, solitons and related topological solutions to field equations, gauge theories, gravitational theory, and even string theory. Free solutions manual available for lecturers at www.wiley-
Geometry, Topology and Physics

Emphasizing the applications of differential geometry to gauge theories in particle physics and general relativity, this work will be of special interest for researchers in applied mathematics or theoretical physics.

A Course in Modern Mathematical Physics

Differential geometry plays an increasingly important role in modern theoretical physics and applied mathematics. This textbook gives an introduction to geometrical topics useful in theoretical physics and applied mathematics, covering: manifolds, tensor fields, differential forms, connections, symplectic geometry, actions of Lie groups, bundles, spinors, and so on. Written in an informal style, the author places a strong emphasis on developing the understanding of the general theory through more than 1000 simple exercises, with complete solutions or detailed hints. The book will prepare readers for studying modern treatments of Lagrangian and Hamiltonian mechanics, electromagnetism, gauge fields, relativity and gravitation. Differential Geometry and Lie Groups for Physicists is well suited for courses in physics, mathematics and engineering for advanced undergraduate or graduate students, and can also be used for active self-study. The required mathematical background knowledge does not go beyond the level of standard introductory undergraduate mathematics courses.

Relativity and Geometry

Suitable for advanced undergraduates and graduate students of mathematics as well as for physicists, this unique monograph and self-contained treatment constitutes an introduction to modern techniques in differential geometry. 1995 edition.

Geometry and Topology

These notes deal with an area that lies at the crossroads of mathematics and physics and rest primarily on the pioneering work of Vaughan Jones and Edward Witten, who related polynomial invariants of knots to a topological quantum field theory in 2+1 dimensions.

Geometry of String Theory Compactifications

"String theory is a leading candidate for the unification of universal forces and matter, and one of its most striking predictions is the existence of small additional dimensions that have escaped detection so far. This book focuses on the geometry of these dimensions, beginning with the basics of the theory, the mathematical properties of spinors, and differential geometry. It further explores advanced techniques at the core of current research, such as G-structures and generalized complex geometry. Many significant classes of solutions to the theory's equations are studied in detail, from special holonomy and Sasaki-Einstein manifolds to their more recent generalizations involving fluxes for form fields. Various explicit examples are discussed, of interest to graduates and researchers"--
Topology and Geometry in Physics

Can artificial intelligence learn mathematics? The question is at the heart of this original monograph bringing together theoretical physics, modern geometry, and data science. The study of Calabi-Yau manifolds lies at an exciting intersection between physics and mathematics. Recently, there has been much activity in applying machine learning to solve otherwise intractable problems, to conjecture new formulae, or to understand the underlying structure of mathematics. In this book, insights from string and quantum field theory are combined with powerful techniques from complex and algebraic geometry, then translated into algorithms with the ultimate aim of deriving new information about Calabi-Yau manifolds. While the motivation comes from mathematical physics, the techniques are purely mathematical and the theme is that of explicit calculations. The reader is guided through the theory and provided with explicit computer code in standard software such as SageMath, Python and Mathematica to gain hands-on experience in applications of artificial intelligence to geometry. Driven by data and written in an informal style, The Calabi-Yau Landscape makes cutting-edge topics in mathematical physics, geometry and machine learning readily accessible to graduate students and beyond. The overriding ambition is to introduce some modern mathematics to the physicist, some modern physics to the mathematician, and machine learning to both.

Twistor Geometry and Field Theory

Publisher Description

Applicable Differential Geometry

This book provides a working knowledge of those parts of exterior differential forms, differential geometry, algebraic and differential topology, Lie groups, vector bundles and Chern forms that are essential for a deeper understanding of both classical and modern physics and engineering. Included are discussions of analytical and fluid dynamics, electromagnetism (in flat and curved space), thermodynamics, the Dirac operator and spinors, and gauge fields, including Yang-Mills, the Aharonov-Bohm effect, Berry phase and instanton winding numbers, quarks and quark model for mesons. Before discussing abstract notions of differential geometry, geometric intuition is developed through a rather extensive introduction to the study of surfaces in ordinary space. The book is ideal for graduate and advanced undergraduate students of physics, engineering or mathematics as a course text or for self study. This third edition includes an overview of Cartan's exterior differential forms, which previews many of the geometric concepts developed in the text.

Applications of Contact Geometry and Topology in Physics

This introductory graduate level text provides a relatively quick path to a special topic in classical differential geometry: principal bundles. While the topic of principal bundles in differential geometry has become classic, even standard, material in the modern graduate mathematics curriculum, the unique approach taken in this text presents the material in a way that is intuitive for both students of mathematics and of physics. The goal of this book is to present important, modern geometric ideas in a form readily accessible to students and researchers in both the physics and mathematics communities, providing each with an understanding and appreciation of the language and ideas of the other.
Lectures on Kähler Geometry

Application of the concepts and methods of topology and geometry have led to a deeper understanding of many crucial aspects in condensed matter physics, cosmology, gravity and particle physics. This book can be considered an advanced textbook on modern applications and recent developments in these fields of physical research. Written as a set of largely self-contained extensive lectures, the book gives an introduction to topological concepts in gauge theories, BRST quantization, chiral anomalies, supersymmetric solitons and noncommutative geometry. It will be of benefit to postgraduate students, educating newcomers to the field and lecturers looking for advanced material.

The Geometry and Physics of Knots

Volume 1 introduces and systematically develops the calculus in a first detailed exposition of this technique which provides shortcuts for some very tedious calculations.

Introduction to Mathematical Physics

A thoroughly revised introduction to non-commutative geometry.

Spinors and Space-Time: Volume 1, Two-Spinor Calculus and Relativistic Fields

Presents the state of the art in PDEs, including the latest research and short courses accessible to graduate students.

An Introduction to Noncommutative Differential Geometry and Its Physical Applications

An introduction to geometrical topics used in applied mathematics and theoretical physics.

Geometrical Methods of Mathematical Physics

First published in 1922 and based on lectures delivered in May 1921, Albert Einstein's The Meaning of Relativity offered an overview and explanation of the then new and controversial theory of relativity. The work would go on to become a monumental classic, printed in numerous editions and translations worldwide. Now, The Formative Years of Relativity introduces Einstein's masterpiece to new audiences. This beautiful volume contains Einstein’s insightful text, accompanied by important historical materials and commentary looking at the origins and development of general relativity. Hanoch Gutfreund and Jürgen Renn provide fresh, original perspectives, placing Einstein's achievements into a broader context for all readers. In this book, Gutfreund and Renn tell the rich story behind the early reception, spread, and consequences of Einstein’s ideas during the formative years of general relativity in the late 1910s and 1920s. They show that relativity's meaning changed radically throughout the nascent years of its development, and they describe in detail the transformation of
Einstein’s work from the esoteric pursuit of one individual communicating with a handful of colleagues into the preoccupation of a growing community of physicists, astronomers, mathematicians, and philosophers. This handsome edition quotes extensively from Einstein’s correspondence and reproduces historical documents such as newspaper articles and letters. Inserts are featured in the main text giving concise explanations of basic concepts, and short biographical notes and photographs of some of Einstein’s contemporaries are included. The first-ever English translations of two of Einstein’s popular Princeton lectures are featured at the book’s end.

**Geometric Algebra for Physicists**

This book provides a comprehensive account of a modern generalisation of differential geometry in which coordinates need not commute. This requires a reinvention of differential geometry that refers only to the coordinate algebra, now possibly noncommutative, rather than to actual points. Such a theory is needed for the geometry of Hopf algebras or quantum groups, which provide key examples, as well as in physics to model quantum gravity effects in the form of quantum spacetime. The mathematical formalism can be applied to any algebra and includes graph geometry and a Lie theory of finite groups. Even the algebra of $2 \times 2$ matrices turns out to admit a rich moduli of quantum Riemannian geometries. The approach taken is a `bottom up` one in which the different layers of geometry are built up in succession, starting from differential forms and proceeding up to the notion of a quantum `Levi-Civita` bimodule connection, geometric Laplacians and, in some cases, Dirac operators. The book also covers elements of Connes’ approach to the subject coming from cyclic cohomology and spectral triples. Other topics include various other cohomology theories, holomorphic structures and noncommutative D-modules. A unique feature of the book is its constructive approach and its wealth of examples drawn from a large body of literature in mathematical physics, now put on a firm algebraic footing. Including exercises with solutions, it can be used as a textbook for advanced courses as well as a reference for researchers.

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